1 Conventional Kolsky-bar (Hopkinson Bar)

A conventional Kolsky-bar is an apparatus in experimental mechanics for the study of properties of materials undergoing high strain rates.

A conventional Kolsky-bar device, schematically shown in Figure 1, consists of a striker, an incident bar, and a transmission bar. The specimen is sandwiched between the incident bar and the transmission bar. The impact of the striker on the end of the incident bar generates elastic waves in both the striker and the incident bar. The elastic wave in the incident bar is called incident wave. It travels through the incident bar to the specimen. Because of the mismatch of mechanical impedances between the bars and the specimen, part of the incident wave is reflected back into the incident bar as a reflected wave and the rest of the incident wave transmits through the specimen into the transmission bar as a transmitted wave. The three wave signals are sensed by the strain gages mounted in the middle of the incident and the transmission bar.

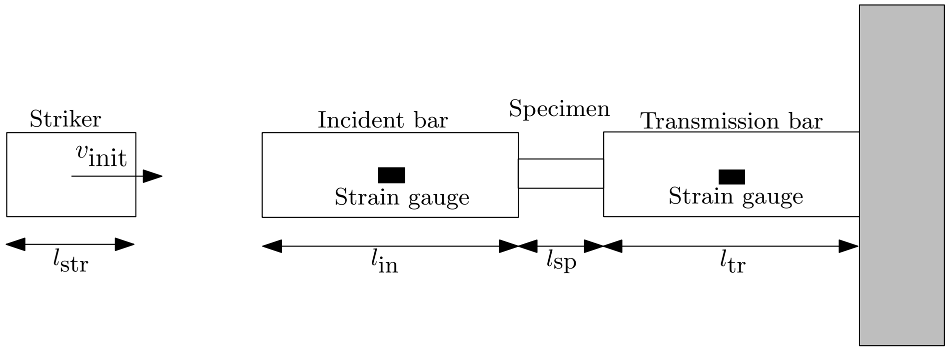

Figure 1: Conventional Kolsky-bar

In the given case, the same material is used for the striker, the incident bar, and the transmission bar. They are described by a cross section A, a Young's Modulus E, a density ρ, and a mechanical impedance Z. During the experiment, the material stays always in its elastic domain. The strain gages measure the longitudinal deformation of the incident wave $\varepsilon_i(t)$, of the reflected wave $\varepsilon_r(t)$, and of the transmitted wave $\varepsilon_t(t)$.

The material of the specimen is characterized by a Young's Modulus E_{sp}, a density ρ_{sp}, a mechanical impedance Z_{sp}, and cross section A_{sp}.

Question 1

Using the shock polar as well as the x-t diagram, determine the time of loss of contact between the striker and the incident bar (the time at which there is no force at the contact interface) as well as the state of stress in the incident bar.

Question 2

There exists a condition ensuring that the measured signal of the incident wave in the incident bar is not overcovered by the signal of the reflected wave. What is this condition? Illustrate the signal measured by the strain gage on the incident bar with respect to time (for $Z_{sp} < Z$).

Question 3

Considering equilibrium of the specimen, determine stress $\sigma_{sp}(t)$, strain rate $\dot{\varepsilon}_{sp}(t)$ and strain $\varepsilon_{sp}(t)$ knowing the measured strain $\varepsilon_i(t)$, $\varepsilon_r(t)$, and $\varepsilon_t(t)$.

Correction

Question 1

The initial state of the striker is called state 1 and is described by an initial velocity $v_1 = v_{init}$ and no stress $\sigma_1 = 0$. The initial state of the incident bar is called state 2 and is described by no velocity $v_2 = 0$ and no stress $\sigma_2 = 0$.

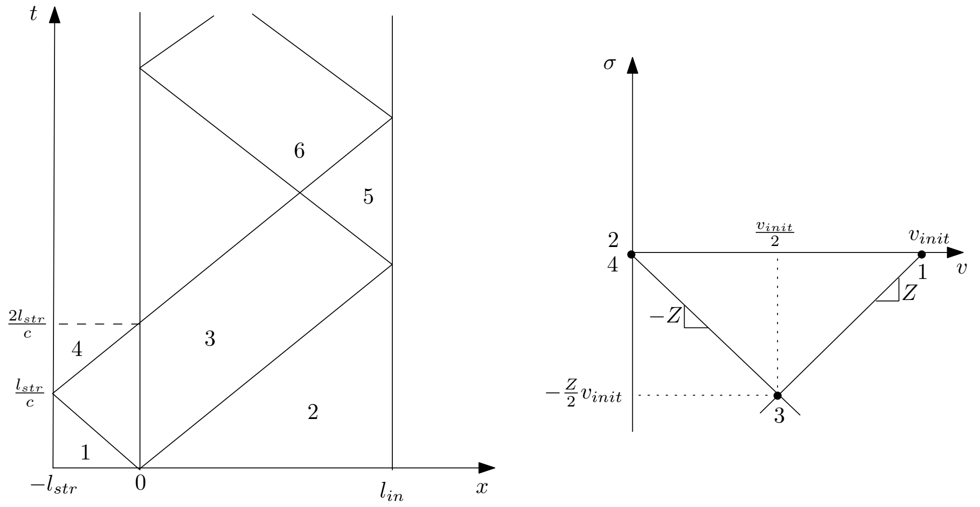

Figure 2: Solution 1

The impact causes a third state that is spreading out from the impact interface in both directions. Using the shock polar, it can be concluded that $v_3 = \frac{v_{init}}{2}$ and $\sigma_3 = -\frac{Z}{2}v_{init}$. This shock wave is reflected at the left boundary of the striker and causes therefore a forth state. Using again the shock polar, we know that $v_4 = 0$ and $\sigma_4 = 0$.

It therefore can be conclude that the time of loss of contact is when condition 4 arrives at the interface, which is the case at $t = \frac{2l_{str}}{c}$.

Question 2

If we want to avoid overcovering of wave signals at the strain gages, the configuration of the experiment has to be designed so that the strain gages measure the following suite of conditions: 2 then 3 then 4 then 6 and so on.

As a first step, we compute the point l_c at which condition 4 and 5 impact on each other and create condition 6:

$$\frac{2l_{str}}{c} + \frac{l_c}{c} = \frac{l_{in}}{c} + \frac{l_{in} - l_c}{c}$$
$$\rightarrow \quad l_c = l_{in} - l_{str}$$

In order to avoid overcovering of the two signals, this point at wich condition 4 and 5 meet each other has to be to the right of the position of the strain gage $l_{sg} = \frac{l_{in}}{2}$. Therefore:

$$l_{in} - l_{str} > \frac{l_{in}}{2}$$
$$\rightarrow l_{str} < \frac{l_{in}}{2}$$

It can be observed that the incident wave is compressive whereas the reflected wave is a tension wave. Furthermore, the amplitude of the reflection wave is smaller than the amplitude of the incident wave because a part of the incident wave is transmitted by the specimen.

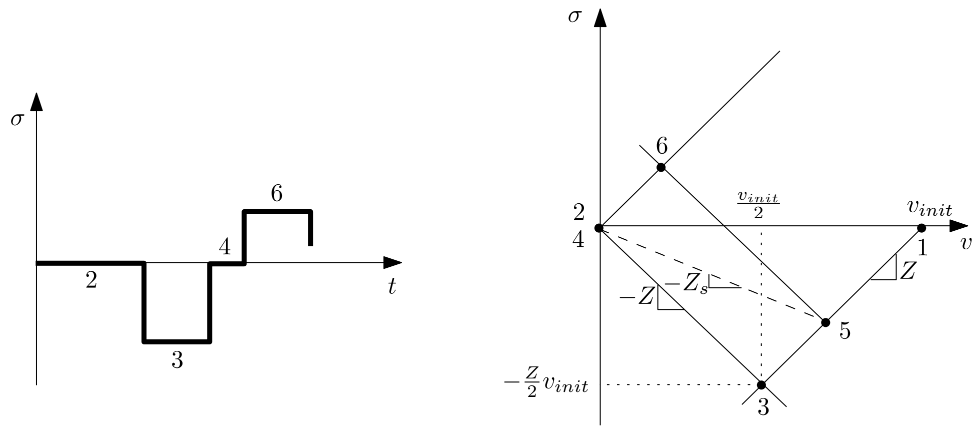

Figure 3: Solution 2

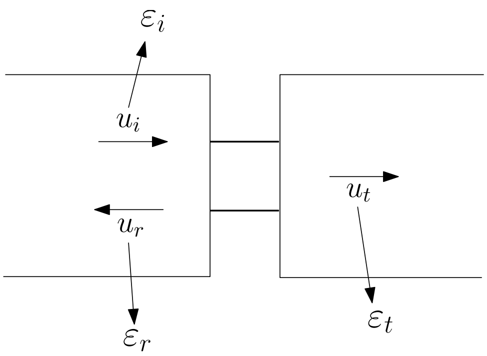

Figure 4: Solution 3

Question 3

Any displacement field and hence strain field describes the displacement or the strain at a given point l_p at any moment t, by $\varepsilon(t) = \varepsilon(l_p \pm ct)$ depending on the direction of the wave. Therefore the strain at the interface of the incident bar and the specimen is $\varepsilon(t) = \varepsilon_i((l_{in} - l_{sg}) - ct)$ and $\varepsilon(t) = \varepsilon_r((l_{in} - l_{sg}) + ct)$. The strain at the interface of the specimen and the transmission bar is $\varepsilon(t) = \varepsilon((l_{in} + l_{sp} - l_{sg}) - ct)$.

Consider first the force at the interface between the incident bar and the specimen:

$$F_{in} = \sigma_{in} A = E \varepsilon_{in} A = EA[\varepsilon_i(t) + \varepsilon_r(t)]$$

Consider now the force at the interface between the specimen and the transmission bar:

$$F_{tr} = \sigma_{tr} A = E \varepsilon_{tr} A = EA[\varepsilon_t(t)]$$

Given equilibrium of the specimen:

$$\begin{aligned} & \sigma_{in} A = \sigma_{sp} A_{sp} = \sigma_{tr} A \\ \rightarrow \quad & \varepsilon_t(t) = \varepsilon_i(t) + \varepsilon_r(t) \quad \text{and} \\ & \sigma_{sp} = \frac{\sigma_{in} A}{A_{sp}} = \frac{EA}{A_{sp}} \varepsilon_t(t) \end{aligned}$$

The strain rate is then:

$$\dot{\varepsilon}_{sp} = \frac{v_{tr} - v_{in}}{l_{sp}} = \frac{-c}{l_{sp}}(\varepsilon_t(t) + \varepsilon_r(t) - \varepsilon_i(t)) = \frac{-2c}{l_{sp}} \varepsilon_r(t)$$

Therefore the strain can be computed as:

$$\varepsilon_{sp}(t) = -2 \frac{c}{l_{sp}} \int_0^t \varepsilon_r(t)\, dt$$